

Fast Track 4

Mathematics Workbook

W. Geldof

Written for able Year 12 students aiming to achieve
Level 2 standards and continue onto a Level 3 course

© W. Geldof 2011
© Sigma Publications Ltd. 2011



This book is copyright.
Under the Copyright Act 1994, no part of this publication may be reproduced,
stored in a retrieval system, or transmitted, in any form, or by any means,
electronic, mechanical, photocopying, recording, or otherwise,
without the prior written permission of the publisher or its agent.

Licences for limited reprographic reproduction by
educational institutions may be obtained from
Copyright Licensing Limited,
P.O. Box 33-1488, Takapuna 0740,
North Shore City, New Zealand
Ph 0800 480 271

Printed and bound in Malaysia through
Bookpac Production Services, Singapore.

Published by Sigma Publications Ltd
P.O. Box 15-248 Tauranga 3144, New Zealand
Phone 0800 274 462, Fax 0800 274 460
Email : mail@sigmapublications.co.nz
Website : www.sigmapublications.co.nz

The logo for Sigma Publications, featuring a stylized orange 'S' inside a circle followed by the word 'sigma' in a bold, lowercase sans-serif font, with 'PUBLICATIONS' in a smaller, uppercase sans-serif font underneath.

ISBN 978-1-877567-04-9

2 Contents

AS 2.1 - Coordinate Geometry

Internal Assessment - 2 credits

Using Grid Paper	5
Line Segments	6
Gradient	7
Drawing Straight Lines	8
Points on a Line	9
Writing Equations 1	10
Writing Equations 2	11
Writing Equations 3	12
Intersections 1	13
Intersections 2	14
Linear Modelling	15
Location	16
Geometry Problems	17
AS 2.1 - Coordinate Geometry - Test	18

AS 2.2 - Graphical Methods

Internal Assessment - 4 credits

Relations 1	19
Relations 2	20
Functions	21
Using a Graphing Calculator	22
Polynomials 1	23
Polynomials 2	24
Polynomials 3	25
Rectangular Hyperbolas 1	26
Rectangular Hyperbolas 2	27
Exponential Curves	28
Logarithmic Curves	29
Transformations of Functions	30
Mixed Non-linear Graphs	31
Piecewise Functions	32
Solving Problems 1	33
Solving Problems 2	34
Solving Problems 3	35
Graphing Trig Functions 1	36
Graphing Trig Functions 2	37
Graphing Trig Functions 3	38
Radian Measure	39
Graphs with Radians	40
Trig Graphs on the Graphing Calculator	41
Writing Trig Equations	42
Solving Trig Equations with a GC	43
Modelling 1	44
Modelling 2	45
AS 2.2 - Graphical Methods - Test	46 & 47

AS 2.3 - Sequences and Series

Internal Assessment - 2 credits

Sequences	48
Terms of an AP	49
Terms of a GP	50
Using a Spreadsheet	51
Sums of an AP	52
Sums of a GP	53
Using Algebra	54
Σ -Notation	55
Sum to Infinity	56
APs in Context	57
GPs in Context	58
Growth and Decay 1	59
Growth and Decay 2	60
Solving Problems	61
AS 2.3 - Sequences and Series - Test	62

AS 2.4 - Trigonometry

Internal Assessment - 3 credits

Right-Angled Triangles 1	63
Right-Angled Triangles 2	64
New Definitions	65
Two Solutions	66
The Area Rule	67
The Sine Rule	68
The Cosine Rule	69
The Rules in Context 1	70
The Rules in Context 2	71
Sine Rule Revisited	72
Proving Formulas	73
Radians	74
Sectors and Arcs	75
Segments	76
Surveying and Navigation	77
AS 2.4 - Trigonometry - Test	78

AS 2.6 - Algebraic Methods

External Assessment - 4 credits

Simplifying Expressions	79
Expanding Brackets 1	80
Expanding Brackets 2	81
Factorising 1	82
Factorising 2	83
Factorising 3	84
Rational Expressions 1	85
Rational Expressions 2	86
Linear Equations	87

Introduction

AS 2.6 - continued

Equations with Fractions	88
Solution Sets	89
Linear Inequations	90
Quadratic Equations 1	91
Quadratic Equations 2	92
Roots of Quadratic Equations	93
Non-linear Inequations	94
Solving Equations with a Graphing Calculator.....	95
Looking at Graphs.....	96
Equations with Constants 1	97
Equations with Constants 2	98
Writing a Formula	99
Using Linear Equations	100
Using Quadratic Equations	101
Index Rules 1	102
Index Rules 2	103
Index Equations	104
More Expressions and Equations	105
Introducing the Log	106
Simplifying Log Expressions	107
Solving Equations with Logs	108
Problem Solving	109
AS 2.6 - Expressions - Test	110
AS 2.6 - Equations - Test	111
AS 2.6 - Applications - Test	112

AS 2.7 - Calculus Methods

External Assessment - 5 credits

Rates of Change	113
Investigating Gradients	114
Graphing Gradient Functions 1	115
Graphing Gradient Functions 2	116
Graphing Gradient Functions 3	117
Graphing Gradient Functions 4	118
The Differentiation Rule 1	119
The Differentiation Rule 2	120
Using Calculus	121
Other Notations	122
Increasing, Decreasing or Stationary	123
Turning Points	124
Tangents to a Curve	125
Modelling	126
Kinematics Part 1	127
Optimisation	128
Mixed Problems 1	129
Mixed Problems 2	130
Anti-Differentiation	131
Sketching the Original Curve	132

AS 2.7 - continued

Evaluating the Constant	133
A Case Study	134
Kinematics Part 2	135
AS 2.7 - Calculus Methods - Test	136 & 137

AS 2.9 - Statistical Methods

Internal Assessment - 4 credits

Populations	138
Sampling Methods 1	139
Sampling Methods 2	140
Comparing Sampling Methods	141 & 142
Evaluating Sampling Methods	143
Variation	144
Sample Statistics 1	145
Sample Statistics 2	146
Sample Statistics 3	147
Calculating Statistics with a GC	148
Shape of a Distribution 1	149
Shape of a Distribution 2	150
Variability Due to Sampling Method	151
Variability Due to Sample Size	152-154
Reliable Inferences	155
An Interval Estimate for Population Median 1	156
An Interval Estimate for Population Median 2	157
Comparing Populations	158
Case Study - 'Tall Poppies'	159-161
Case Study - 'Carrying the Load'	162-164
Assessment Requirements	165
AS 2.9 - Statistical Methods - Investigation	166

AS 2.12 - Probability Methods

External Assessment - 4 credits

Vocabulary and Notations	167
Absolute Risk and Relative Risk 1	168
Absolute Risk and Relative Risk 2	169
Absolute Risk and Relative Risk 3	170
Introduction to Tree Diagrams	171
Using Tree Diagrams 1	172
Using Tree Diagrams 2	173
Using Tree Diagrams 3	174
Playing Games	175
Mean, Standard Deviation and Proportion	176
Approximately Bell Shaped	177
The Normal Distribution 1	178
The Normal Distribution 2	179
The Normal Distribution 3	180
The Normal Distribution 4	181

AS 2.12 - continued

Normal Distribution Problems 1	182
Normal Distribution Problems 2	183
Normal Distribution Problems 3	184
AS 2.12 - Probability Methods - Test	185-186

AS 2.14 - Systems of Equations

Internal Assessment - 2 credits

Finding Points of Intersection	187
Substitution	188
Lines and Curves	189
Logo Design	190
Elimination	191
Simultaneous Equations with the GC	192
Using Simultaneous Equations 1	193
Using Simultaneous Equations 2	194
Using Simultaneous Equations 3	195
AS 2.14 - Systems of Equations - Test.....	196

Answers (removable)	197 - 215
Achievement Standards	216 - 220

Statistics Database Available on Sigma's Website

In the Statistics chapter of this workbook you will be asked to investigate a database of information on New Zealand athletes. You can download the complete database from Sigma's website. You will require a computer with internet access and the microsoft spreadsheet program *Excel*.

Download instructions :

- 1] Go to Sigma's website at www.sigmapublications.co.nz and click on 'Secondary Maths Books' in the side panel menu.
- 2] Click on the cover picture of '*Fast Track 4*' at the bottom of this page.
- 3] This will take you to the '*Fast Track 4*' page. At the bottom of this page you will find the NZ Olympic Athletes spreadsheet listed.
- 4] To download the database, click on the spreadsheet.

About this Book

A Choice of Standards

This mathematics workbook '*Fast Track 4*' prepares students to pass NCEA Level 2 Achievement Standards with merit or excellence. The work is drawn from levels 6 and 7 of the New Zealand Mathematics and Statistics curriculum.

There are a total of 14 Achievement Standards available at Level 2 Mathematics. It is expected that a full-time Level 2 student will study between 5 and 7 of these standards (about 20-24 credits). The selected standards that students sit at this level must cover the three strands of the curriculum : Number and Algebra, Geometry and Statistics. Most schools will make this selection based on the ability and interests of their students.

Since no Level 2 student is expected to do all 14 standards, it would be a waste of money and paper to ask able maths students to purchase a workbook containing enough material to cover the full range of 14 standards. Therefore, after consultation with Heads of Maths departments, I have selected the nine most popular achievement standards being taught to the vast majority of *able* Level 2 maths students.

'*Fast Track 4*' provides revision work for the following Achievement Standards :

AS 2.1 - Coordinate Geometry	2 credits
AS 2.2 - Graphical Methods	4 credits
AS 2.3 - Sequences & Series	2 credits
AS 2.4 - Trigonometry	3 credits
AS 2.6 - Algebraic Methods	4 credits
AS 2.7 - Calculus	5 credits
AS 2.9 - Statistics	4 credits
AS 2.12 - Probability	4 credits
AS 2.14 - Systems of Equations	2 credits

[black credits are internally assessed, orange credits indicate externally assessed standards.]

It is expected that students will use a range of technology throughout the year (graphing calculator, spreadsheet). This book includes some basic instructions for the *Casio fx9750 GII* graphic calculator and the *Microsoft Excel* spreadsheet. However, these explanations are brief and more detailed information can be found in the graphic calculator's instruction booklet and in *Excel's* help menu.

I wish you well with your studies this year.

Wiesje Geldof